NOI '01 P4 - Applications of Arctangent

Time limit: 0.18s **Memory limit:** 64M

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The inverse tangent function can be expressed as an infinite series, as shown below:

$$\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \quad (0 \le x \le 1) \tag{1}$$

It is commonly known that the inverse tangent function can be used to compute π. For example, an easy way to compute π is using the method:

$$\begin{aligned} \pi &= 4\arctan(1) \\ &= 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots\right) \end{aligned} \tag{2}$$

Of course, this method is rather inefficient. We can apply the tangent angle sum identity:

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)} \tag{3}$$

After some simple manipulation, the following is obtained:

$$\arctan(p) + \arctan(q) = \arctan\left(\frac{p+q}{1-pq}\right) \tag{4}$$

Using this identity, let $p = \frac{1}{2}$ and $q = \frac{1}{3}$, then $\frac{p+q}{1-pq} = 1$. Therefore:

$$\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right) = \arctan\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}\right) = \arctan(1)$$

Using the inverse tangents of $\frac{1}{2}$ and $\frac{1}{3}$ to calculate $\arctan(1)$, the speed is drastically improved.

We take equation (4) and write it in the following form:

$$\arctan\left(\frac{1}{a}\right) = \arctan\left(\frac{1}{b}\right) + \arctan\left(\frac{1}{c}\right)$$

where a, b, and c are each positive integers.

The problem is, for a given a $(1 \le a \le 60\,000)$, find the value of $b + c$. It is guaranteed that for any a there will always exist an integer solution. If there are multiple solutions, you are required to find the minimum value of $b + c$.

Input Specification

The input consists of a single positive integer a, where $1 \le a \le 60\,000$.

Output Specification

The output should contain a single integer, the value of $b + c$.

Sample Input

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1
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Sample Output

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5
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Problem translated to English by **Alex**.