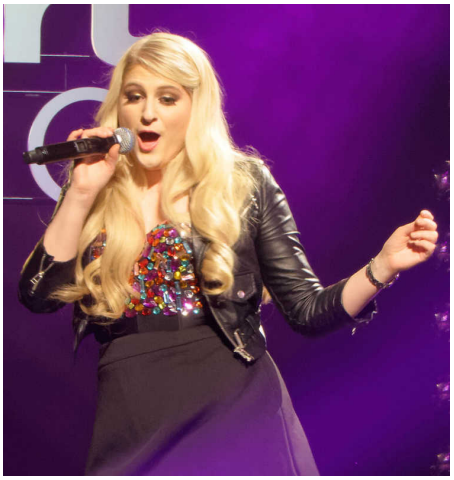
ICPC NAQ 2015 A - All About That Base

Time limit: 0.6s **Memory limit:** 1G

ICPC North America Qualifier 2015, Problem A



The *base* (or *radix*) of a positional numeral system is the number of symbols that can be used to represent a number in that system. The base 10 system (also known as decimal) uses 10 distinct symbols: $0, 1, \ldots, 9$. For example, we interpret the number $72\,345$ as:

$$7 \times 10^4 + 2 \times 10^3 + 3 \times 10^2 + 4 \times 10^1 + 5 \times 10^0.$$

This example illustrates that in base 10 the symbol at place $P \geq 0$ (starting from the right) is multiplied by 10^P to get its value. More generally, in base B we use B symbols to represent $0, \ldots, B-1$, and the symbol at the P^{th} place is multiplied by B^P to get its value.

Other bases commonly used in computation include base 2 (or binary, using symbols 0 and 1), base 8 (or octal, using symbols 0–7), and base 16 (or hexadecimal, using symbols 0–9 and a–f). In bases higher than 10, letters represent the higher values. Thus in hexadecimal a–f represent the decimal values 10–15, and in bases ≥ 36 the letter z represents the decimal value 35.

Your job is to determine the bases in which given arithmetic expressions are valid. We define an expression as *valid* in base B if two conditions are true. First, all the operands used are interpretable in base B as having values in the decimal range $[1, 2^{32}-1]$. Second, the expression is true. Any arbitrary expression might be valid in zero, one, or more bases. In this problem we will only consider bases 1–36, where base 1 is unary.

Note that following the convention listed above, unary would consist of a single symbol: 0. In this problem, unary numbers use the symbol 1 rather than 0 (think "tally marks"). E.g., 111 in unary is equivalent to the decimal number 3 and 1111111 in unary is equivalent to the decimal number 7.

Input Specification

Input for this problem starts with a line containing an integer $0 \leq N \leq 20$. The following N lines each contain an arithmetic expression with the following form:

$$X \text{ op } Y = Z$$

where X, Y, and Z are positive, whole numbers consisting of 1 to 100 symbols from the set 0–9 and a–z, and op is one of the four operators `+`, `-`, `*`, `/`. For each statement there is at least one base $1 \leq B \leq 36$ such that X, Y, and Z can all be interpreted in base B as having values in the decimal range $[1, 2^{32}-1]$.

Output Specification

For each expression, list the bases in which the expression is valid (sorted in ascending base order) or the word `invalid` if the expression is not valid in any of the bases 1–36. Use symbols 1–9, then a–z, then 0 to represent bases 1–36 (with the last symbol, 0, representing base 36).

Sample Input

```
8
6ef + d1 = 7c0
3 / 2 = 1
444 / 2 = 222
10111 * 11 = 1000101
10111 * 11 = 111221
5k - 1z = 46
1111111111 - 1111111 = 111
2048 - 512 = 1536
```

Sample Output

```
g
invalid
56789abcdefghijklmnopqrstuvwxyz0
2
3456789abcdefghijklmnopqrstuvwxyz0
invalid
1
a
```