

# JOI '23 Open P3 - Garden

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**Time limit:** 2.0s    **Memory limit:** 1G

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JOI Kingdom is a mysterious kingdom which has a boundless expanse of territory. JOI-kun, the king of JOI Kingdom, is planning to cut a part of the territory and make his garden.

The territory of JOI Kingdom is considered as a sufficiently large 2-dimensional grid. The grid is paved with square cells from the top to the bottom and from the left to the right. There is a cell, which is the origin of the coordinates. Let  $(x, y)$  denote the cell one arrives at when one moves from the origin to the right direction for the distance of  $x$  cells and to the upward direction for the distance of  $y$  cells. Here, the left direction for the distance of  $a$  cells means the right direction for the distance of  $-a$  cells. Similarly, the downward direction for the distance of  $a$  cells means the upward direction for the distance of  $-a$  cells.

Some artworks are placed on the territory. The artworks are classified into two types, **Type A** and **Type B**, according to the way to be placed in the territory.

- There are  $N$  kinds of artworks of type A. An artwork of  $i$ -th kind ( $1 \leq i \leq N$ ) is placed on every cell of the form  $(P_i + kD, Q_i + lD)$ , where  $k, l$  are integers.
- There are  $M$  kinds of artworks of type B. An artwork of  $j$ -th kind ( $1 \leq j \leq M$ ) is placed on every cell of the form  $(R_j + kD, y)$ , where  $k, y$  are integers, or of the form  $(x, S_j + lD)$ , where  $l, x$  are integers.

Note that a cell may contain several artworks of different kinds.

JOI-kun is planning to choose a rectangular region on the grid to make a garden. In other words, he will choose 4 integers  $a, b, c, d$ . Then the cells of the form  $(x, y)$ , where  $x, y$  are integers satisfying  $a \leq x \leq b, c \leq y \leq d$ , will constitute JOI-kun's garden. Since JOI-kun likes to see artworks of many kinds, for any of the  $N + M$  kinds of artworks, JOI-kun's garden should contain at least one artwork of that kind. On the other hand, the citizens of JOI Kingdom will be angry if JOI-kun plans to make a too large garden. Therefore, JOI-kun wants to minimize the number of cells in the garden so that the above condition is satisfied.

Write a program which, given information of artworks, calculates the minimum number of cells in JOI-kun's garden.

## Input Specification

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Read the following lines from Standard Input.

$N M D$

$P_1 Q_1$

$P_2 Q_2$

$\vdots$

$P_N Q_N$

$R_1 S_1$

$R_2 S_2$

$\vdots$

$R_M S_M$

## Output Specification

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Write one line to the standard output. The output should contain the minimum number of cells in JOI-kun's garden.

## Constraints

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- $N \geq 1$ .
- $M \geq 1$ .
- $N + M \leq 500\,000$ .
- $1 \leq D \leq 5\,000$ .
- $0 \leq P_i < D$  ( $1 \leq i \leq N$ ).
- $0 \leq Q_i < D$  ( $1 \leq i \leq N$ ).
- $0 \leq R_j < D$  ( $1 \leq j \leq M$ ).
- $0 \leq S_j < D$  ( $1 \leq j \leq M$ ).
- Given values are all integers.

## Subtasks

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1. (15 points)  $M \leq 8$ .
2. (6 points)  $D \leq 10$ ,  $N + M \leq 5000$ .
3. (8 points)  $D \leq 50$ ,  $N + M \leq 5000$ .
4. (16 points)  $D \leq 100$ ,  $N + M \leq 5000$ .
5. (30 points)  $N + M \leq 5000$ .
6. (25 points) No additional constraints.

## Sample Input 1

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```
2 1 5
1 4
2 2
0 0
```

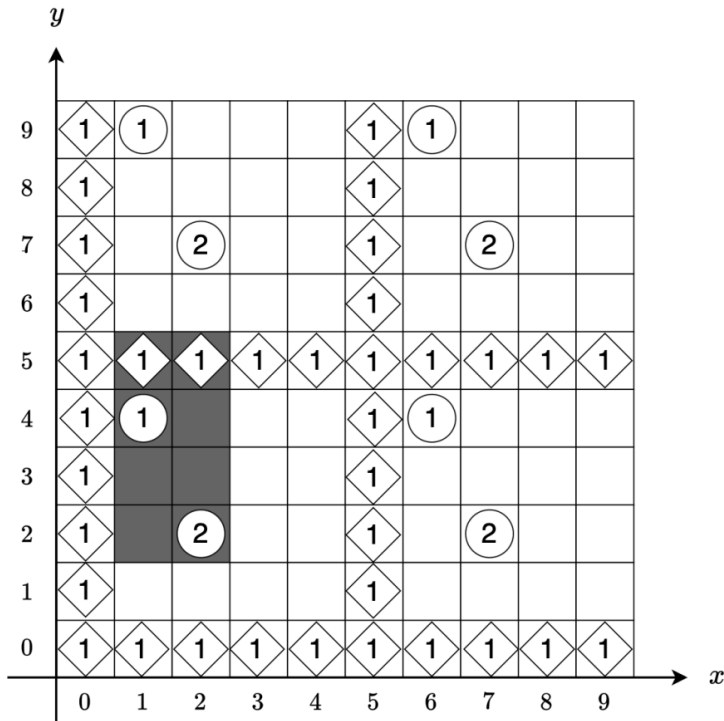
## Sample Output 1

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```
8
```

## Explanation for Sample 1

The following figure describes the cells  $(x, y)$ , where  $x, y$  are integers satisfying  $0 \leq x < 10, 0 \leq y < 10$ , in the territory of JOI Kingdom.



In this figure, circles and diamond shapes are artworks of type A and B, respectively. An integer in a circle or a diamond shape describes the kind of the artwork. If JOI-kun chooses  $a = 1, b = 2, c = 2, d = 5$ , JOI-kun's garden is a black rectangular region. In this case, JOI-kun's garden has at least one artwork of any of the 3 kinds of artworks. The number of cells in the garden is 8. Since there is no garden which satisfies the condition and which has smaller number of cells, output 8.

This sample input satisfies the constraints of all the subtasks.

## Sample Input 2

```
3 4 100
20 26
81 56
20 3
58 71
74 82
95 61
95 61
```

## Sample Output 2

2840

## Explanation for Sample 2

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This sample input satisfies the constraints of Subtasks 1, 4, 5, 6.

## Sample Input 3

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```
5 7 5000
1046 365
4122 1166
4009 2896
1815 4065
4372 1651
2382 123
1475 836
3313 4005
2579 568
4300 4867
1050 3214
3589 4653
```

## Sample Output 3

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10543092

## Explanation for Sample 3

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This sample input satisfies the constraints of Subtasks 1, 5, 6.