

IOI '24 P4 - Hieroglyphs

Time limit: 1.0s **Memory limit:** 1G

A team of researchers is studying the similarities between sequences of hieroglyphs. They represent each hieroglyph with a non-negative integer. To perform their study, they use the following concepts about sequences.

For a fixed sequence A , a sequence S is called a **subsequence** of A if and only if S can be obtained by removing some elements (possibly none) from A .

The table below shows some examples of subsequences of a sequence $A = [3, 2, 1, 2]$.

Subsequence	How it can be obtained from A
[3, 2, 1, 2]	No elements are removed.
[2, 1, 2]	[3, 2, 1, 2]
[3, 2, 2]	[3, 2, 1, 2]
[3, 2]	[3, 2, 1, 2] or [3, 2, 1, 2]
[3]	[3, 2, 1, 2]
[]	[3, 2, 1, 2]

On the other hand, [3, 3] or [1, 3] are not subsequences of A .

Consider two sequences of hieroglyphs, A and B . A sequence S is called a **common subsequence** of A and B if and only if S is a subsequence of both A and B . Moreover, we say that a sequence U is a **universal common subsequence** of A and B if and only if the following two conditions are met:

- U is a common subsequence of A and B .
- Every common subsequence of A and B is also a subsequence of U .

It can be shown that any two sequences A and B have at most one universal common subsequence.

The researchers have found two sequences of hieroglyphs A and B . Sequence A consists of N hieroglyphs and sequence B consists of M hieroglyphs. Help the researchers compute a universal common subsequence of sequences A and B , or determine that such a sequence does not exist.

Implementation details

You should implement the following procedure.

```
std::vector<int> ucs(std::vector<int> A, std::vector<int> B)
```

- A : array of length N describing the first sequence.
- B : array of length M describing the second sequence.
- If there exists a universal common subsequence of A and B , the procedure should return an array containing this sequence. Otherwise, the procedure should return $[-1]$ (an array of length 1, whose only element is -1).
- This procedure is called exactly once for each test case.

Constraints

- $1 \leq N \leq 100\,000$
- $1 \leq M \leq 100\,000$
- $0 \leq A[i] \leq 200\,000$ for each i such that $0 \leq i < N$
- $0 \leq B[j] \leq 200\,000$ for each j such that $0 \leq j < M$

Subtasks

Subtask	Score	Additional Constraints
1	3	$N = M$; each of A and B consists of N distinct integers between 0 and $N - 1$ (inclusive)
2	15	For any integer k , (the number of elements of A equal to k) plus (the number of elements of B equal to k) is at most 3.
3	10	$A[i] \leq 1$ for each i such that $0 \leq i < N$; $B[j] \leq 1$ for each j such that $0 \leq j < M$
4	16	There exists a universal common subsequence of A and B .
5	14	$N \leq 3000$; $M \leq 3000$
6	42	No additional constraints.

Examples

Example 1

Consider the following call.

```
ucs([0, 0, 1, 0, 1, 2], [2, 0, 1, 0, 2])
```

Here, the common subsequences of A and B are the following: $[], [0], [1], [2], [0, 0], [0, 1], [0, 2], [1, 0], [1, 2], [0, 0, 2], [0, 1, 0], [0, 1, 2], [1, 0, 2]$ and $[0, 1, 0, 2]$.

Since $[0, 1, 0, 2]$ is a common subsequence of A and B , and all common subsequences of A and B are subsequences of $[0, 1, 0, 2]$, the procedure should return $[0, 1, 0, 2]$.

Example 2

Consider the following call.

```
ucs([0, 0, 2], [1, 1])
```

Here, the only common subsequence of A and B is the empty sequence $[\]$. It follows that the procedure should return an empty array $[\]$.

Example 3

Consider the following call.

```
ucs([0, 1, 0], [1, 0, 1])
```

Here, the common subsequences of A and B are $[\]$, $[0]$, $[1]$, $[0, 1]$ and $[1, 0]$. It can be shown that a universal common subsequence does not exist. Therefore, the procedure should return $[-1]$.

Sample Grader

Input format:

```
N M
A[0] A[1] ... A[N-1]
B[0] B[1] ... B[M-1]
```

Output format:

```
T
R[0] R[1] ... R[T-1]
```

Here, R is the array returned by `ucs` and T is its length.

Attachment Package

The sample grader along with sample test cases are available [here](#).