

IOI '23 P1 - Closing Time

Time limit: 1.0s **Memory limit:** 1G

Hungary is a country with N cities, numbered from 0 to $N - 1$.

The cities are connected by $N - 1$ bidirectional roads, numbered from 0 to $N - 2$. For each j such that $0 \leq j \leq N - 2$, road j connects city $U[j]$ and city $V[j]$ and has length $W[j]$, that is, it allows one to travel between the cities in $W[j]$ units of time. Each road connects two different cities, and each pair of cities is connected by at most one road.

A **path** between two distinct cities a and b is a sequence p_0, p_1, \dots, p_t of distinct cities, such that:

- $p = a$,
- $p = b$,
- for each i ($0 \leq i < t$), there is a road connecting cities p_i and p_{i+1} .

It is possible to travel from any city to any other city by using the roads, that is, there exists a path between every two distinct cities. It can be shown that this path is unique for each pair of distinct cities.

The **length** of a path p_0, p_1, \dots, p_t is the sum of the lengths of the t roads connecting consecutive cities along the path.

In Hungary, many people travel to attend the Foundation Day festivities in two major cities. Once the celebrations are over, they return to their homes. The government wants to prevent the crowd from disturbing the locals, so they plan to lock down all cities at certain times. Each city will be assigned a non-negative closing time by the government. The government has decided that the sum of all closing times must not be more than K . More precisely, for every i between 0 and $N - 1$, inclusive, the closing time assigned to city i is a nonnegative integer $c[i]$. The sum of all $c[i]$ must not be greater than K .

Consider a city a and some assignment of closing times. We say that a city b is **reachable** from city a if and only if either $b = a$, or the path p_0, \dots, p_t between these two cities (so in particular $p_0 = a$ and $p_t = b$) satisfies the following conditions:

- the length of the path p_0, p_1 is at most $c[p_1]$, and
- the length of the path p_0, p_1, p_2 is at most $c[p_2]$, and
- ...
- the length of the path $p_0, p_1, p_2, \dots, p_t$ is at most $c[p_t]$.

This year, the two main festival sites are located in city X and city Y . For each assignment of closing times, the **convenience score** is defined as the sum of the following two numbers:

- The number of cities reachable from city X .
- The number of cities reachable from city Y .

Note that if a city is reachable from city X and reachable from city Y , it counts twice towards the convenience score.

Your task is to compute the maximum convenience score that can be achieved by some assignment of closing times.

Implementation Details

You should implement the following procedure.

```
int max_score(int N, int X, int Y, long long K, std::vector<int> U, std::vector<int> V,
std::vector<int> W)
```

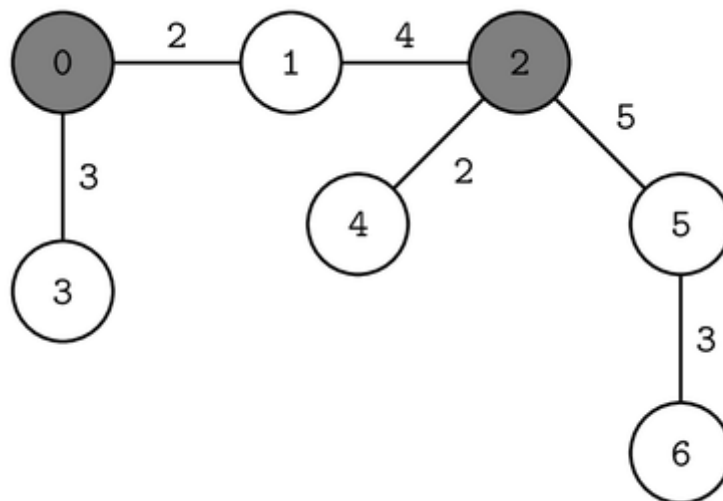
- N : the number of cities.
- X, Y : the cities with main festival sites.
- K : the upper bound on the sum of closing times.
- U, V : arrays of length $N - 1$ describing road connections.
- W : array of length $N - 1$ describing road lengths.
- This procedure should return the maximum convenience score that can be achieved by some assignment of closing times.
- This procedure may be called **multiple times** in each test case.

Example

Consider the following call:

```
max_score(7, 0, 2, 10,
[0, 0, 1, 2, 2, 5], [1, 3, 2, 4, 5, 6], [2, 3, 4, 2, 5, 3])
```

This corresponds to the following road network:



Suppose the closing times are assigned as follows:

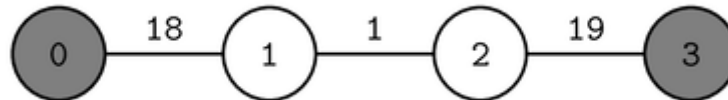
City	0	1	2	3	4	5	6
Closing time	0	4	0	3	2	0	0

Note that the sum of all closing times is 9, which is not more than $K = 10$. Cities 0, 1, and 3 are reachable from city X ($X = 0$), while cities 1, 2, and 4 are reachable from city Y ($Y = 2$). Therefore, the convenience score is $3 + 3 = 6$. There is no assignment of closing times with convenience score more than 6, so the procedure should return 6.

Also, consider the following call:

```
max_score(4, 0, 3, 20, [0, 1, 2], [1, 2, 3], [18, 1, 19])
```

This corresponds to the following road network:



Suppose the closing times are assigned as follows:

City	0	1	2	3
Closing time	0	1	19	0

City 0 is reachable from city X ($X = 0$), while cities 2 and 3 are reachable from city Y ($Y = 3$). Therefore, the convenience score is $1 + 2 = 3$. There is no assignment of closing times with convenience score more than 3, so the procedure should return 3.

Constraints

- $2 \leq N \leq 200\,000$
- $0 \leq X < Y < N$
- $0 \leq K \leq 10^{18}$
- $0 \leq U[j] < V[j] < N$ (for each j such that $0 \leq j \leq N - 2$)
- $1 \leq W[j] \leq 10^6$ (for each j such that $0 \leq j \leq N - 2$)
- It is possible to travel from any city to any other city by using the roads.
- $S_N \leq 200\,000$, where S_N is the sum of N over all calls to `max_score` in each test case.

Subtasks

We say that a road network is linear if road i connects cities i and $i + 1$ (for each i such that $0 \leq i \leq N - 2$).

1. (8 points) The length of the path from city X to city Y is greater than $2K$.
2. (9 points) $S_N \leq 50$, the road network is linear.
3. (12 points) $S_N \leq 500$, the road network is linear.
4. (14 points) $S_N \leq 3\,000$, the road network is linear.
5. (9 points) $S_N \leq 20$
6. (11 points) $S_N \leq 100$

7. (10 points) $S_N \leq 500$
8. (10 points) $S_N \leq 3\,000$
9. (17 points) No additional constraints.

Sample Grader

Let C denote the number of scenarios, that is, the number of calls to `max_score`. The sample grader reads the input in the following format:

- line 1: C

The descriptions of C scenarios follow.

The sample grader reads the description of each scenario in the following format:

- line 1: $N X Y K$
- line $2 + j$ ($0 \leq j \leq N - 2$): $U[j] V[j] W[j]$

The sample grader prints a single line for each scenario, in the following format:

- line 1: the return value of `max_score`

Attachment Package

The sample grader along with sample test cases are available [here](#).