CCC '22 S3 - Good Samples

Canadian Computing Competition: 2022 Stage 1, Senior #3

You are composing music for the Cool Clarinet Competition (CCC). You have been instructed to make a piece of music with exactly N notes. A note is represented as a positive integer, indicating the pitch of the note.

We call a non-empty sequence of consecutive notes in the piece a sample. For instance, $(3, 4, 2)$, $(1, 2, 3, 4, 2)$, and (4) are samples of $1, 2, 3, 4, 2$. Note that $(1, 3)$ is not a sample of $1, 2, 3, 4, 2$. We call two samples different if they start or end at a different position in the piece.

We call a sample *good* if no two notes in the sample have the same pitch.

The clarinet players are picky in two ways. First, they will not play any note with pitch higher than M. Second, they want a piece with exactly K good samples.

Can you construct a piece to satisfy the clarinet players?

Input Specification

The first and only line of input will contain 3 space-separated integers, N, M, and K.

The following table shows how the available 15 marks are distributed.

Marks Awarded	Bounds on N	Bounds on M	Bounds on K
3 marks	$1 \leq N \leq 16$	$M = 2$	$1 \leq K \leq 1\,000$
3 marks	$1 \leq N \leq 10^6$	$M = 2$	$1 \leq K \leq 10^{18}$
4 marks	$1 \leq N \leq 10^6$	$M = N$	$1 \leq K \leq 10^{18}$
5 marks	$1 \leq N \leq 10^6$	$1 \leq M \leq N$	$1 \leq K \leq 10^{18}$

Output Specification

If there is a piece of music that satisfies the given constraints, output N integers between 1 and M, representing the pitches of the notes of the piece of music. If there is more than one such piece of music, any such piece of music may be outputted.

Otherwise, output -1.

Sample Input 1

```
3 2 5
```

Sample Output 1

```
1 2 1
```

Explanation of Output for Sample Input 1

Notice that the piece is composed of $N = 3$ notes, each of which is one of $M = 2$ possible pitches, 1 and 2. That piece of music has a total of 6 samples, but only $K = 5$ good samples: (1), $(1, 2)$, (2), $(2, 1)$, (1). Notice that the two good samples of (1) are different since they start at two different positions.

Note that the piece 212 is the only other valid output for this input.

One example of an output that would be incorrect is 323, since it has notes with pitches larger than 2. Another incorrect output would be 112, since it only has four good samples: (1), (1), (2), and $(1, 2)$.

Sample Input 2

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5 5 14
```

Sample Output 2

```
1 5 3 2 1
```

Explanation of Output for Sample Input 2

The 14 good samples are: (1), $(1, 5)$, $(1, 5, 3)$, $(1, 5, 3, 2)$, (5), $(5, 3)$, $(5, 3, 2)$, $(5, 3, 2, 1)$, (3), $(3, 2)$, $(3, 2, 1)$, (2), $(2, 1)$, (1).

Sample Input 3

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5 5 50
```

Sample Output 3

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-1
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Explanation of Output for Sample Input 3

There are no pieces with 5 notes that can produce 50 different good samples.