CCC '21 S5 - Math Homework

Time limit: 3.0s **Memory limit:** 1G

Canadian Computing Competition: 2021 Stage 1, Senior #5

Your math teacher has given you an assignment involving coming up with a sequence of N integers $A_1, \ldots, A_N$, such that $1 \leq A_i \leq 1\,000\,000\,000$ for each i.

The sequence A must also satisfy M requirements, with the i^{th} one stating that the GCD (Greatest Common Divisor) of the contiguous subsequence $A_{X_i}, \ldots, A_{Y_i}$ $(1 \leq X_i \leq Y_i \leq N)$ must be equal to Z_i. Note that the GCD of a sequence of integers is the largest integer d such that all the numbers in the sequence are divisible by d.

Find *any* valid sequence A consistent with all of these requirements, or determine that no such sequence exists.

Input Specification

The first line contains two space-separated integers, N and M. The next M lines each contain three space-separated integers, X_i, Y_i, and Z_i $(1 \leq i \leq M)$.

The following table shows how the available 15 marks are distributed.

Subtask	N	M	Z_i
3 marks	$1 \leq N \leq 2\,000$	$1 \leq M \leq 2\,000$	$1 \leq Z_i \leq 2$ for each i
4 marks	$1 \leq N \leq 2\,000$	$1 \leq M \leq 2\,000$	$1 \leq Z_i \leq 16$ for each i
8 marks	$1 \leq N \leq 150\,000$	$1 \leq M \leq 150\,000$	$1 \leq Z_i \leq 16$ for each i

Note: an additional test case worth 1 point was added to prevent unintended solutions from passing.

Output Specification

If no such sequence exists, output the string `Impossible` on one line. Otherwise, on one line, output N space-separated integers, forming the sequence $A_1, \ldots, A_N$. If there are multiple possible valid sequences, any valid sequence will be accepted.

Sample Input 1

```
2 2
1 2 2
2 2 6
```

Output for Sample Input 1

```
4 6
```

Explanation of Output for Sample Input 1

If $A_1 = 4$ and $A_2 = 6$, the GCD of $[A_1, A_2]$ is 2 and the GCD of $[A_2]$ is 6, as required. Please note that other outputs would also be accepted.

Sample Input 2

```
2 2
1 2 2
2 2 5
```

Output for Sample Input 2

```
Impossible
```

Explanation of Output for Sample Input 2

There exists no sequence $[A_1, A_2]$ such that the GCD of $[A_1, A_2]$ is 2 and the GCD of $[A_2]$ is 5.