CCC '19 S2 - Pretty Average Primes

Time limit: 1.0s Memory limit: 1G

Canadian Computing Competition: 2019 Stage 1, Senior #2

For various given positive integers N > 3, find two primes, A and B such that N is the average (mean) of A and B. That is, N should be equal to $\frac{A+B}{2}$.

Recall that a *prime number* is an integer P > 1 which is only divisible by 1 and P. For example, 2, 3, 5, 7, 11 are the first few primes, and 4, 6, 8, 9 are not prime numbers.

Input Specification

The first line of input is the number T ($1 \le T \le 1000$), which is the number of test cases. Each of the next T lines contain one integer N_i ($4 \le N_i \le 1000000$, $1 \le i \le T$).

For 6 of the available 15 marks, all $N_i < 1\,000$.

Output Specification

The output will consist of T lines. The i^{th} line of output will contain two integers, A_i and B_i , separated by one space. It should be the case that $N_i = \frac{A_i + B_i}{2}$ and that A_i and B_i are prime numbers.

If there are more than one possible A_i and B_i for a particular N_i , output any such pair. The order of the pair A_i and B_i does not matter.

It will be the case that there will always be at least one set of values A_i and B_i for any given N_i .

Sample Input

4		
8		
4		
7		
21		

Sample Output

Explanation of Possible Output for Sample Input

Notice that:

$$egin{aligned} 8&=rac{3+13}{2},\ 4&=rac{5+3}{2},\ 7&=rac{7+7}{2},\ 21&=rac{13+29}{2}. \end{aligned}$$

It is interesting to note, that we can also write

$$8 = \frac{5+11}{2}$$

$$21 = \frac{5+37}{2} = \frac{11+31}{2} = \frac{19+23}{2}$$

$$7 = \frac{3+11}{2}$$

and so any of these pairs could have also been used in output. There is no pairs of primes other than 3 and 5 which average to the value of 4.

Footnote

You may have heard about *Goldbach's conjecture*, which states that every even integer greater than 2 can be expressed as the sum of two prime numbers. There is no known proof, yet, so if you want to be famous, prove that conjecture (after you finish the CCC).

This problem can be used to help verify that conjecture, since every even integer can be written as 2N, and your task is to find two primes A and B such that 2N = A + B.