CCC '18 S4 - Balanced Trees

Time limit: 1.0s **Memory limit:** 256M

Canadian Computing Competition: 2018 Stage 1, Senior #4

Trees have many fascinating properties. While this is primarily true for trees in nature, the concept of trees in math and computer science is also interesting. A particular kind of tree, a *perfectly balanced tree*, is defined as follows.

Every perfectly balanced tree has a positive integer *weight*. A perfectly balanced tree of weight 1 always consists of a single node. Otherwise, if the weight of a perfectly balanced tree is w and $w \geq 2$, then the tree consists of a root node with branches to k subtrees, such that $2 \leq k \leq w$. In this case, all k subtrees must be completely identical, and be perfectly balanced themselves.

In particular, all k subtrees must have the same weight. This common weight must be the maximum integer value such that the sum of the weights of all k subtrees does not exceed w, the weight of the overall tree. For example, if a perfectly balanced tree of weight 8 has 3 subtrees, then each subtree would have weight 2, since $2 + 2 + 2 = 6 \leq 8$.

Given N, find the number of perfectly balanced trees with weight N.

Input Specification

The input will be a single line containing the integer N $(1 \leq N \leq 10^9)$.

For 5 of the 15 marks available, $N \leq 1\,000$.

For an additional 2 of the 15 marks available, $N \leq 50\,000$.

For an additional 2 of the 15 marks available, $N \leq 10^6$.

Output Specification

Output a single integer, the number of perfectly balanced trees with weight N.

Sample Input 1

```
4
```

Sample Output 1

```
3
```

Explanation for Sample Output 1

One tree has a root with four subtrees of weight 1; a second tree has a root with two subtrees of weight 2; the third tree has a root with three subtrees of weight 1.

Sample Input 2

```
10
```

Sample Output 2

```
13
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