An Animal Contest 5 P2 (Hard Version) - Permutations & Primes

Time limit: 3.0s **Memory limit:** 256M

This problem is a harder version of Permutations & Products. In this version, you can only ask questions where $|i-j|$ is prime.

Larry the magical panda is bored of eating bamboo cookies, so he challenges you to a game. He has a permutation of $1, 2, \ldots, N$ which he calls A, and you have to guess the permutation by asking questions. The questions work as follows:

- You will give Larry two distinct indices i and j $(1 \leq i, j \leq N, i \neq j)$ such that $|i-j|$ is prime
- Larry will respond with the result of $A_i \times A_j$

Larry allows you to ask at most $N-1$ questions. Can you guess the permutation and win the game?

Constraints

$4 \leq N \leq 10^5$

Interaction

This is an interactive problem, where you and the judge exchange information back-and-forth to solve the problem.

At first, you should read in a line containing the integer N.

You will then start the interaction by proceeding with your questions. Each question should be formatted as `? i j` followed by a `\n` character, with $1 \leq i, j \leq N$, $i \neq j$, and $|i-j|$ is prime. In response, you will be given $A_i \times A_j$ on its own line.

If you believe you have the solution, you may output `!` followed by a space-separated list of N integers $A_1, A_2, \ldots, A_N$, the permutation A. You must terminate your program immediately after performing this operation. Note that this operation does not count towards the limit of $N-1$ questions.

If at any point you attempt an invalid question (such as an invalid output format or a prohibited pair of indices), or you exceed the limit of $N-1$ questions, the interactor will respond with -1. You should terminate your program immediately after receiving this feedback to receive a Wrong Answer verdict, or you may receive an arbitrary verdict. If the final list you output is incorrect, you will receive a Wrong Answer verdict. Otherwise, you will receive a verdict of Accepted for the corresponding test case.

Please note that you may need to flush `stdout` after each operation, or interaction may halt. In C++, this can be done with `fflush(stdout)` or `cout << flush` (depending on whether you use `printf` or `cout`). In Java, this can be done with `System.out.flush()`. In Python, you can use `sys.stdout.flush()`.

Sample Interaction

`>>>` denotes your output. Do not print this out.

```
5
>>> ? 3 5
10
>>> ? 1 4
4
>>> ? 4 2
3
>>> ! 4 3 2 1 5
```